International Journal of Theoretical Physics, Vol. 14, No. 4 (1975), pp. 217–235

Strongly Interacting Particle Physics

PAUL K. S. SUH

Grumman Aerospace Research Center, Bethpage, New York 11714, *U.S.A.*

Received: 25 *November* 1974

Abstract

A dynamical strongly interacting particle theory is presented. The theory explicitly exhibits the structures of particle states and provides intriguing insights into their working mechanisms. Agreement of the results with the experimental observations is excellent.

1. *Introduction*

Despite the inherent differences of the interactions involved, the situation in the hypothetical atomic states where the electromagnetic interaction constant becomes of the order of 1 with nonvanishing photon mass might become comparable in a number of aspects to that of the baryon states in the strongly interacting particle physics. A multitudinous number of K and $\bar{K}$ mesons in a nucleon N (or $\bar{N}$) field[1] (carrying the basic units of the strongly interacting particle quantum numbers), with due support from π, η, ρ and other mesons that could in turn be formed from the particles mentioned above, may then participate in the formation of baryon states.

Experimental observations, in fact, indicate that the high-energy particle interaction occurs predominantly at a small momentum transfer, favoring the multiperipheral interaction. As discussed in the appendix, a new degree of uncertainty may also arise from the very nature of strong interaction to help accommodate numerous constituent particles in the formation of baryon states. On the basis of the energy relation discussed in Section 2, a pointedly fitting conclusion then is that the isotopic spin vectors of the particles participating in the formation of baryon states may couple to yield large mass level shifts.

The isotopic spin vector interaction diagram is thus introduced to identify interaction schemes that provide a simple means of predicting particle states

[1] If the N-$\bar{N}$ pairs and others also participate in the interaction at a very high-energy domain, the total cross section may increase.

in a remarkable one-to-one agreement with observation.[2] A few basic rules are prescribed for the determination of angular momenta and parity of the baryon states also in agreement with observation, yielding further insights into the particle structure. The phenomenon of meson resonances is then considered.

All the interaction constants present in the theory are conjectured to be derivable from a single interaction parameter. The theory emerges in an increasingly cohesive form and gives such deep insights into the workings of the strongly interacting particle physics that the author believes the probability of the agreement's being accidental should be quite small.

2. *Mass Level Formula*

Consider a simple Yukawa potential energy

$$V(r) = -g^2 \frac{e^{-\mu r}}{r} \tag{2.1}$$

which, if nature is truly simple in essence, may give some insight into the structure of strong interaction. The corresponding kinetic energy for a bound state in a virial theorem approximation is

$$T \approx \frac{r}{2} \frac{dV(r)}{dr} \approx g^2 \frac{\mu r + 1}{2r} e^{-\mu r} \tag{2.2}$$

yielding, at the effective interaction radius $r \approx 1/\mu$,

$$\langle E \rangle \approx \langle T + V \rangle \approx \langle g^2 \frac{\mu r - 1}{2r} e^{-\mu r} \rangle \approx 0 \tag{2.3}$$

Analysis of the particle states in this paper do uphold the general validity of the energy relation in various partial wave states (see the appendix). A $\bar{K}$ meson participating in the $\bar{K}$-N "dynamical interaction" to form baryon states in the approximation of energy relation [2.3] would contribute $M_{\bar{K}} + e_{\bar{K}}$ to the mass level, where the adjustable constant $e_{\bar{K}}$ empirically turns out to be negligibly small.

The possible physical implication of the phenomenon of forming stable hyperons (Λ, Σ) out of the $\bar{K}$-N interaction has been explored by Suh (1970). The succinct explanation of the interaction is that the $\bar{K}$ meson field equation with a nucleon source is suggestive of a "quasidynamical interaction" at an imaginary eigen-momentum $i\mu_{\bar{K}N}$ that, when evaluated at the reduced $\bar{K}$-N

[2] The theory, originally proposed in 1959 (Suh, 1960, 1968), has consistently predicted particle states [including the Ω_0 (1680) state before the advent of the $SU(3)$ theory] to be observed subsequently. Since one can see how easily particle states can be predicted in the theory, however, this paper will not engage in making claims for such achievement.

Compton wavelength $1/\mu_{\bar{K}N}$, yields an energy contribution[3] of

$$\hat{M}_{\bar{K}} \approx [M_{\bar{K}}^2 - \mu_{\bar{K}N}^2]^{\frac{1}{2}} + \langle V_{\bar{K}} \rangle \approx 235 \text{ MeV} \tag{2.4}$$

It is now tempting to estimate the isotopic spin vector coupling terms by

$$V_I(r) \approx \frac{1}{M_1 M_2} \frac{1}{r} \frac{dV}{dr} (\vec{I}_1 \cdot \vec{I}_2) \tag{2.5}$$

where the situation, if indeed relevant, is no doubt grossly simplified. Here, $M_{1,2}$ and $I_{1,2}$ are masses and isotopic spins, respectively, while V represents the relevant Yukawa potential energy of the interacting particles. The π-N and $\bar{K}$-N isotopic spin vector coupling terms are then given, by using $g_{\pi-N}^2 \approx 15$ and $g_{\bar{K}-N}^2 \approx 2$, as

$$\langle V_I \rangle_{\pi N} \approx 2G\,(\vec{I}_N \cdot \vec{I}_\pi) \qquad \text{and} \qquad \langle V_I \rangle_{\bar{K}N} \approx 2f_n (\vec{I}_N \cdot \vec{I}_{\bar{K}}) \tag{2.6}$$

where $G \approx M_\pi$ and $f_n \approx 35$ MeV. The isotopic spin vector coupling in the π-N interaction is thus much stronger than that in the $\bar{K}$-N interaction, suggesting that in a baryon state where the $\bar{K}$-N interaction is assisted by an involvement of pions, the pion fields should first be coupled to the core N-field so that the resultant field can be coupled subsequently to the $\bar{K}$ meson field.

It can also be surmised in analogy with atomic physics that, while the $\bar{K}$-N interaction with a smaller isotopic spin vector coupling constant may allow the $\bar{K}$ meson isotopic spin vectors to couple to the core N-field isotopic spin vector in such a manner that all may be added together (in parallel),[4] the π-N interaction with a larger isotopic spin vector coupling constant causes every pion isotopic spin vector to couple individually to the core N-field isotopic spin vector.

Even if the simple K-N interaction itself is not attractive, in a way comparable to the ionic bonding force in molecules, with enough $\bar{K}$ mesons (and possibly other mesons as well) on hand, the K meson may experience a transient field attractive enough to participate in the dynamical interaction for the formation of baryon states. Even the positive strangeness baryon states can now be visualized, although such states would require an interaction structure more complex than the simple K-N interaction. It is apparent that these positive strangeness baryon states are very unstable and that the number of K mesons should not exceed that of $\bar{K}$ mesons in the formation of comparatively stable baryon states.

The K and $\bar{K}$ mesons participating in the formation of a baryon state should, however, be prevented from outright annihilation by evading direct coupling between them, unless the possible K-$\bar{K}$ meson correlation produces other heavier mesons to interact. It is then evident that for the K and $\bar{K}$

[3] The heuristically derived (without being necessarily geared to the validity of the overall theory itself) interaction constants in this paper are semiempirical in that they are determined (in the vicinity of those theoretically estimated) by using a few observed baryon mass levels.

[4] The interaction of $\bar{K}$ mesons, being required to be symmetric, would favor having their isotopic spins paralleled.

mesons to be able to participate in the formation of distinctly observable baryon states, the isotopic spin couplings of the $\bar{K}$ mesons should be completely detached from those of the K mesons. As discussed in the appendix and Section 7, the K mesons in the baryon states are also impelled to orbitals larger than those of the $\bar{K}$ mesons, to diminish the probability of outright annihilation not only by avoiding the spatial overlapping of interaction, but also by possible enactment of selection rules on angular momenta.

Possible involvement of two or more K mesons in a baryon state of strangeness S_B requires a mass level of over 2000 MeV for the $S_B = 0$ state and a correspondingly higher mass level for the $S_B < 0$ state. Since extension can be made to more complex particle interaction configurations that yield mass levels above 2000 MeV, this paper limits consideration to nonpositive strangeness baryon states in which only a zero or a one K meson is involved.

The main support of the $(K, \bar{K})$ meson interaction in the formation of baryon states comes from pions and, depending on the interaction structure available, varying numbers of pions become involved. Thus, baryon states are distinguished according to how many pions actively participate in the formation of a baryon state. Such heavier mesons as η and ρ are also observed to contribute to the formation of comparatively higher-energy baryon states. However, the η meson, having a zero isotopic spin, is observed to involve itself only in a quite simple threshold interaction, bearing out the proposed role of isotopic spin couplings in the formation of particle states.

The ρ meson reveals the remarkable characteristic of possibly being able to form itself out of a K-$\bar{K}$ meson pair interaction. This makes it possible to estimate the ρ-meson isotopic spin vector coupling constant to be in the order of that of the $(K, \bar{K})$ meson interaction. Dynamical interaction of these $x = \pi, \eta$, and ρ mesons in the baryon states contribute $M_x + e_x$ to the mass levels, with the empirically determined adjustable constants $e_\pi \approx e_\eta \approx e_\rho \approx 25$ MeV.

As the interaction configuration changes, the interaction constants become modified. For instance, when f_n in equation (2.6) is defined as the coupling constant in an n_π-pion baryon state, empirical values are $f_0 \approx 39$ MeV and $f_{1,2} \approx 35$ MeV. Since the K meson in particular interacts in various angular momentum states, as discussed in Section 7, this near uniformity of the interaction constants f_n for $n = 0, 1$, and 2 is one of the consequences that indicate the appropriateness of the expansible potential approximation discussed in the appendix, which, in turn, supports the general validity of the energy relation (2.3). The empirical π-π meson isotopic spin coupling constant g for nonresonance coupling is small (approximately 13 MeV).

Forego, for the moment, consideration of the baryon states possessing mesons heavier than π and $(K, \bar{K})$ mesons, and consider only up to the two-pion baryon states. Then the above explorations already yield abundant information as to how baryon states can be constructed, yielding the following baryon mass levels:

$$M_B \approx M_N + F_{N\pi} + F_{N\pi\bar{K}} + F_{N\pi K} \tag{2.7}$$

where

$$F_{N\pi} \approx \Sigma(M_\pi + e_\pi + 2\,\vec{I}_N \cdot \vec{I}_\pi G)_{n_\pi} + 2\,\vec{I}_\pi \cdot \vec{I}_\pi g$$
$$F_{N\pi\bar{K}} \approx -\,\bar{S}\hat{M}_{\bar{K}} + 2\,\vec{I}_{N\pi} \cdot \vec{I}_{\bar{K}} f_n, F_{N\pi K} \approx SM_K + 2\,\vec{I}_{N\pi} \cdot \vec{I}_K f_n;$$

$\vec{I}_N$ and $\vec{I}_\pi$ = nucleon core and pion isotopic spin vectors, $\vec{I}_{K,\bar{K}}$ = total isotopic spin vector (added parallel) of the $(K, \bar{K})$ mesons with their strangeness $(S, \bar{S})$, where $S_B = S + \bar{S}$, and

$$\vec{I}_{N\pi} = \vec{I}_N + \Sigma(\vec{I}_\pi)_{n_\pi}, \vec{I}_1 = \vec{I}_{N\pi} + \vec{I}_{\bar{K}}, \vec{I} = \vec{I}_1 + \vec{I}_K \tag{2.7}$$

The baryon states are subject to the condition

$$I + |\bar{S}| \lesssim n_\pi + \tfrac{5}{2} \tag{2.8}$$

when their maximum charges are restricted to $n_\pi + 1$. When any of the $(K, \bar{K})$ mesons in a baryon state depart from the interactions characterized by equation (2.7), as discussed in Section 5, the baryon state is designated to be in an excited state. Involvement of mesons heavier than those considered here in the interaction is discussed in Section 6.

3. *Zero-Pion Baryon States*

For the zero-pion baryon states, equation (2.7) reduces to

$$M_B \approx M_N - \bar{S}\hat{M}_{\bar{K}} + 2\,\vec{I}_N \cdot \vec{I}_{\bar{K}} f_0 + SM_K + 2\,\vec{I}_N \cdot \vec{I}_K f_0 \tag{3.1}$$

Now, under the condition (2.8), consider the $S = 0$ states and introduce the isotopic spin-vector interaction diagrams of Figure 1. The first number of the bracket in the diagram represents the isotopic spin I, and the second, the strangeness S_B of the state. The vector $\vec{I}_{\bar{K}}$ that is added parallel represents the $\bar{K}$ mesons, and the "x" between $\vec{I}_N$ and $\vec{I}_{\bar{K}}$ indicates not only that the

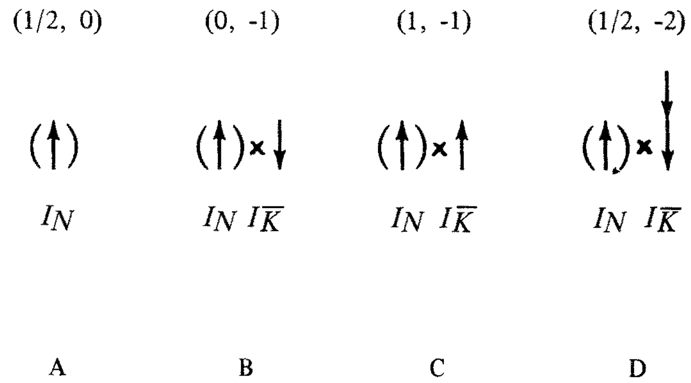

Figure 1–Interaction diagrams $M_{1A,B,C,D}$.

isotopic spin-vector coupling between the two vectors is in operation but also that each of the $\bar{K}$ mesons interacts quasidynamically with the nucleon core, providing an energy contribution of $\hat{M}_{\bar{K}}$.

By using the interaction constants given in Section 2, the mass levels $M_{1B,C,D}$ of the baryon states for the interaction diagrams of Figures 1B, C, and D are calculated to yield

$$M_{1B,C,D} \approx [1115, 1193, 1330] \text{ MeV} \tag{3.2}$$

which agree well with the observed mass levels for the stable baryon states[5] (Λ, Σ, Ξ). A multiplet is formed by successively adding $\bar{K}$ mesons to a baryon state with their isotopic spins along a given direction.

Now consider the $S = 1$ states given in Figure 2, where the vector $\vec{I}_K$ in these diagrams represents the K meson. The connection "..." between $\vec{I}_N$

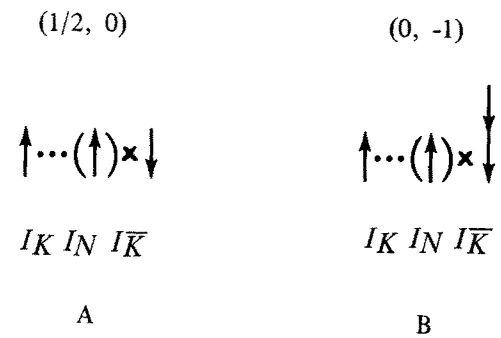

Figure 2–Interaction diagrams $M_{2A,B}$. Spin-flippings of $\vec{I}_K$ and $\vec{I}_{\bar{K}}$ and excitation in these states produce other baryon states.

and $\vec{I}_K$ indicates that the K meson interacts dynamically with the isotopic spin vector coupling in operation, thus yielding

$$M_{2A,B} \approx [1635, 1845] \text{ MeV} \tag{3.3}$$

[$M^*_{2B}(K) \approx 1825$ MeV[6] in the K meson excited baryon state (see Section 5)] which correspond to the observed $N_{\frac{1}{2}}(1650)$ and $Y_0(1835)$ states, respectively (and the excited state to the $Y_0(1820)$ state). A spin flip of the $\vec{I}_K$ vector in Figure 2 now gives

$$M_{2A,B-1} \approx [1550, 1760] \text{ MeV} \tag{3.4}$$

General observation, including nonzero-pion baryon states, reveals that, while it is not outright forbidden, a baryon state in which all the $\vec{I}_\pi$'s, if any,

[5] For experimental data, see recent issues of *Review of Modern Physics* or *Physics Letters*. Note that the term "octet" is a misnomer here.

[6] This $M^*_{2B}(K)$ state gives $F(\frac{5}{2})^+$ interaction, while the M_{2B} state yields a $D(\frac{5}{2})^-$ interaction.

are parallel to $\vec{I}_N$ and both $\vec{I}_K$ and $\vec{I}_{\bar{K}}$ (in the $S \neq 0$ state) are antiparallel to $\vec{I}_N$, appears to encounter some difficulties in forming a well defined state. This situation will be referred to as Anomaly I. An associated phenomenon, to be referred to as Anomaly IA, is also observed when both the $\vec{I}_\pi$ and $\vec{I}_K$ vectors are antiparallel to the I_N vector, its effect becoming apparent in the higher $|S_B|$ states in a given multiplet and especially when the $\vec{I}_{\bar{K}}$ vector is also antiparallel to the I_N vector.

The difficulties manifested by Anomaly IA may be circumvented, as will be shown through examples, either by the K meson excitation when the $\vec{I}_{\bar{K}}$ vector is parallel to $\vec{I}_N$, or often by the process of $K + \bar{K} \to \rho$ (as for the case of Anomaly I) when the $\vec{I}_{\bar{K}}$ vector is also antiparallel to $\vec{I}_N$. Baryon states, when affected by the Anomalies, thus frequently exhibit deviative characteristics from their usual quantum number assignment.

Therefore, as affected by Anomaly I, the M_{2A-1} state is observed as an enhancement shoulder corresponding to the $N_{\frac{1}{2}}(1550)$ state, while the weak M_{2B-1} state appears to be enhanced by the M_{4B} state discussed in Section 4 and is observed as a strong resonance state Y_1 (1765). Finally, the spin-flippings of both the $\vec{I}_K$ and $\vec{I}_{\bar{K}}$ vectors in Figure 2A produce a state of mass level $M_{2A-2} \approx M_{2A}$. The $\Delta I_K = -2$ rule discussed in Section 7, however, gives an S-wave interaction for the K meson in the M_{2A-2} state. The annihilation effect of the S-wave K meson with the quasidynamically interacting S-wave $\bar{K}$ meson diminishes the observability of the M_{2A-2} state. The K meson could, however, interact in an excited state M^*_{7B} as discussed in Section 5. The states considered above represent all of the zero-pion baryon states under the condition (2.8).

4. *One-Pion Baryon States*

Mass levels of the one-pion baryon states are given by

$$M_B \approx M_N + F_{N\pi} + F_{N\pi\bar{K}} + F_{N\pi K} \tag{4.1}$$

where $F_{N\pi} \approx M_\pi + e_\pi + 2\,\vec{I}_N \cdot \vec{I}_\pi G$, $F_{N\pi\bar{K}} \approx -\bar{S}\hat{M}_{\bar{K}} + 2\,\vec{I}_{N\pi} \cdot \vec{I}_K f_1$, and $F_{N\pi K} \approx SM_K + 2\,\vec{I}_{N\pi} \cdot \vec{I}_K f_1$. Therefore, the mass levels for the $S = 0$ states of Figure 3 are

$$M_{3A,B,C,D} \approx [1238, 1385, 1532, 1659] \text{ MeV} \tag{4.2}$$

corresponding to the $N_{\frac{3}{2}}(1238)$, $Y_1(1385)$, $\Xi_{\frac{1}{2}}(1532)$, and $\Omega_0(1680)$ states.

The isotopic spin vectors $\vec{I}_\pi$, $\vec{I}_{\bar{K}}$, and $\vec{I}_K$ in the $S = 1$ state diagrams of Figure 4 can simply be flipped in all possible combinations of directions to form the various baryon states given in the Table, where (+) and (–) indicate that the $\vec{I}_\pi$, $I_{\bar{K}}$ and $\vec{I}_K$ vectors interact parallel and antiparallel, respectively, to the $\vec{I}_N$ vector. Because the $\vec{I}_{\bar{K}}$ vectors in the $M_{4A,B,C-2}$ states are parallel to $\vec{I}_N$ vector, the effect of Anomaly IA turns out to be mild. In fact, as stated in Section 3, only the higher $|S_B|$ members of the multiplet appear to be appreciably affected, favoring the K meson excited states $M^*_{4B,C-2}(K)$, as listed in the Table, to circumvent the difficulties manifested by Anomaly IA.

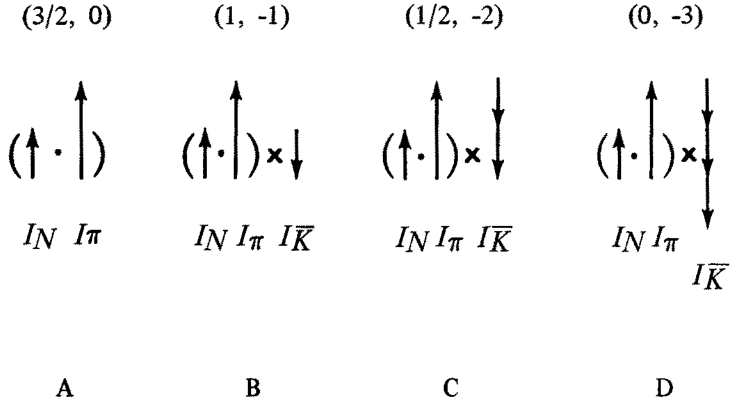

Figure 3—Interaction diagrams $M_{3A,B,C,D}$.

Among the $M_{4A,B,C-4}$ states affected by Anomaly I, only the M_{4A-4} state, by transforming to the ρ-meson baryon state $M_{12A} \approx 1860$ MeV of Figure 12, appears to be observed as the $N_{\frac{1}{2}}(1860)$ state. Although $M_{4A-5} \approx M_{4A-3}$, the M_{4A-5} state is affected by the $\Delta I_K = -2$ rule discussed in Section 7, yielding $P(\frac{1}{2})^+N_{\frac{3}{2}}(1910)$ and $F(\frac{7}{2})^+N_{\frac{3}{2}}(1920)$ states, respectively. A further coherent consequence is observed for the $M_{4A-2} \approx M_{4A}$ case where, while the M_{4A-2} state compares with the $D(\frac{3}{2})^-N_{\frac{1}{2}}(1520)$ state, the M_{4A} state, also affected by the $\Delta I_K = -2$ rule, would not be observable (see Section 7).

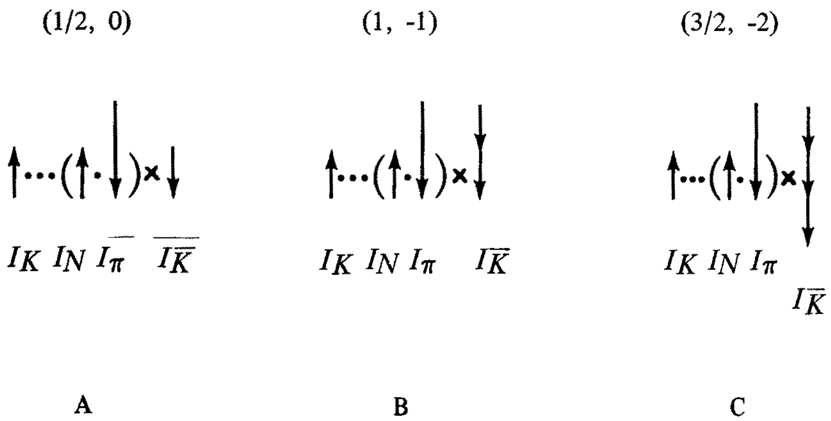

Figure 4—Interaction diagrams $M_{4A,B,C}$. Spin-flippings of $\vec{I}_\pi$, $\vec{I}_K$, and $\vec{I}_{\bar{K}}$, excitation, and transfiguration in these states produce other baryon states.

TABLE 1. Some of the one-pion baryon states produced with various isotopic spin-vector flippings in the interaction diagrams of Figure 4

$\vec{I}_\pi$	$\vec{I}_K$	$\vec{I}_{\bar{K}}$	I	S_B	$\lvert\bar{S}\rvert$	S	M_{cal} (MeV)	M_{exp} (MeV)
			$\frac{1}{2}$	0	1		$M_{4A}(1520)$	Prohibited (see Section 7)
		(–)	1	–1	2		$M_{4B}(1770)$	$Y_1(1765)$
	(+)		$\frac{3}{2}$	–2	3		$M_{4C}(2027)$	Forbidden
			$\frac{1}{2}$	0	1		$M_{4A-1}(1455)$	$N_{\frac{1}{2}}(1460)$
(–)			1	–1	2		$M_{4B-1}(1670)$	$Y_1(1660)$
			$\frac{3}{2}$	–2	3		$M_{4C-1}(1885)$	Forbidden
		(+)	$\frac{1}{2}$	0	1		$M_{4A-2}(1520)$	$N_{\frac{1}{2}}(1520)$
	(–)		0	–1	2		M^*_{4B-2}(K) (1715)	$Y_0(1690)$ (Anomaly IA)
			$\frac{1}{2}$	–2	3		M^*_{4C-2}(K) (1932)	$\Xi_{\frac{1}{2}}(1940)$ (Anomaly IA)
			$\frac{3}{2}$	0	1	1	$M_{4A-3}(1934)$	$N_{\frac{3}{2}}(1920)$
	(+)		1	–1	2		$M_{4B-3}(2080)$	$Y_1(2030)$
			$\frac{1}{2}$	–2	3		$M_{4C-3}(2230)$	$\Xi_{\frac{1}{2}}(2270)$
		(–)	$\frac{1}{2}$	0	1		$M_{4A-4}(1800)$ $\rightarrow M_{12}(1855)$	$N_{\frac{1}{2}}(1860)$; ρ-meson baryon
(+)			0	–1	2		$M_{4B-4}(1950)$	Not favored (Anomaly I)
			$\frac{1}{2}$	–2	3		$M_{4C-4}(2100)$	Not favored (Anomaly I)
	(–)		$\frac{3}{2}$	0	1		$M_{4A-5}(1934)$	$N_{\frac{3}{2}}(1910)$
		(+)	2	–1	2		$M_{4B-5}(2222)$	Forbidden
			$\frac{5}{2}$	–2	3		$M_{4C-5}(2510)$	Forbidden

One of the vector alignments omitted in Table 1 is $\{\vec{I}_\pi, \vec{I}_K, \vec{I}_K\} = \{(-), (-), (-)\}$, yielding

$$M_5 \approx 1600 \text{ MeV} \tag{4.3}$$

of Figure 5. However, as affected by Anomaly IA, the state is likely to transform to a ρ-meson baryon state $M_{12B} \approx 1640$ MeV of Figure 12, thus producing instead an S-wave state, i.e., $S(\frac{1}{2})^- N_{\frac{1}{2}}(1640)$, as observed at this comparatively high energy level. The remaining vector alignment $\{\vec{I}_\pi, \vec{I}_K, \vec{I}_{\bar{K}}\} = \{(+), (+), (+)\}$ is forbidden. The states considered above represent all of the one-pion baryon states under the condition (2.8).

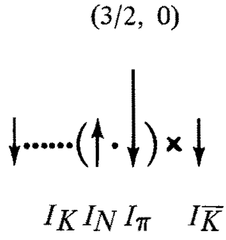

Figure 5—Interaction diagram M_5.

Note that the so-called octet and decuplet baryon states are represented by the simplest possible vector couplings of Figures 1 and 3. The mass level formula of these states can be transformed to be identical with those of the $SU(3)$ theory (Okubo, 1962). However, since the assumption made in the derivation of the $SU(3)$ mass formula is not deduced from any clearly conceived physical mechanism, its underlying meaning is obscure and it is not clear if its mass formula really provides the test of the $SU(3)$ theory.

5. *Excited Baryon States*

As stated in Section 2, the excited baryon states are those in which some of the $(K, \bar{K})$ meson interactions deviate from the interaction scheme defined in equation (2.7). For instance, the $\bar{K}$ meson in the $Y_0(1405)$ state interacts dynamically as represented by Figure 6, giving the mass level

$$M_6^*(\bar{K}) \approx M_N + M_K + B \approx 1405 \text{ MeV} \tag{5.1}$$

in which $B \approx -30$ MeV can be mainly due to the isotopic spin vector coupling.

It is observed that, to produce an effectual $\bar{K}$ meson excitation in a baryon state of comparatively simple interaction configuration, the $\bar{K}$ meson binding is necessarily strong, thus ruling out, for instance, the possible formation of the $I = 1$ state that arise with an $\vec{I}_{\bar{K}}$ vector flip in the M_6^* state. The $x = K$ or $\bar{K}$ meson excited state is denoted throughout this paper by $M_j^*(x)$, the asterisk

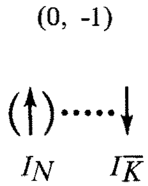

Figure 6—Interaction diagram M_6^*.

indicating an excited state. For example, in the baryon states of Figure 3, a $\bar{K}$ meson may be excited to produce the states

$$M^*_{3B,C}(\bar{K}) \approx [1650, 1800] \text{ MeV} \tag{5.2}$$

The two Y_1 states, M_{4B-1} and $M^*_{3B}(K)$ states, are very close in mass levels, but they belong to two different multiplets, and thus their angular momentum and parity configuration are not identical (see Section 7). The $M^*_{3C}(\bar{K})$ state, on the other hand, corresponds to the $\Xi_{\frac{1}{2}}(1815)$ state. The $\bar{K}$ meson excitation in the M_{4A-2} state, affected by Anomaly IA, gives

$$M^*_{4A-2}(\bar{K}) \approx 1700 \text{ MeV} \tag{5.3}$$

agreeing with the observed $N_{\frac{1}{2}}(1700)$ state.

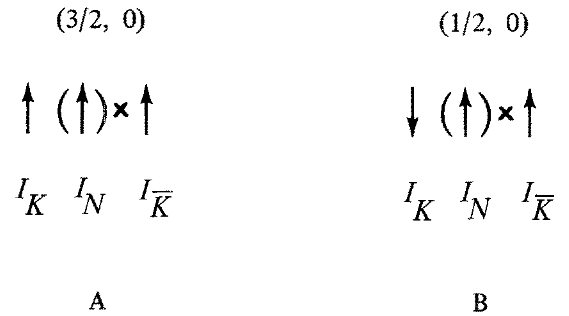

Figure 7—Interaction diagrams $M^*_{7A,B}$.

The excitation of a K meson (whose participation in the formation of baryon states is innately transient, as discussed in Section 2) may largely diminish its isotopic spin coupling strength, simply to contribute, in a crude approximation, a K meson mass to the mass level. This interaction mechanism is manifested by leaving the space between $\vec{I}_N$ (or $\vec{I}_{N\pi}$) and $\vec{I}_K$ vectors empty in the interaction diagrams. The K meson excitation tends to be observed in the higher $|S_B|$ states in a given multiplet and more frequently in the higher multiple pion baryon states (see Sections 3 and 4).

The K meson excitation, for instance, in the forbidden zero-pion baryon state[7] of ($I = \frac{3}{2}$ and $S = |\bar{S}| = 1$) may now produce the M^*_{7A} state, which in turn suggests the M^*_{7B} state (as was also surmised in Section 3), of Figure 7, yielding

$$M^*_{7A}(K) \approx M^*_{7B}(K) \approx 1690 \text{ MeV} \tag{5.4}$$

[7] This state may transfigure to a ρ-meson baryon state $P(\frac{3}{2})^- N_{\frac{3}{2}}(1730)$ that compares well with the observed P_{33} state around 1700 MeV level.

The states not only compare well with the observed mass level for the $N_{\frac{3}{2}}(1690)$ and $N_{\frac{1}{2}}(1690)$ states, but also exhibit, as discussed in Section 7, coherent properties in terms of the angular momenta and parity.

6. *Two–Pion and Heavier Meson Baryon States*

Only a few two-pion baryon states need be considered in the energy-domain concerned in this paper. Mass level for the $I = \frac{5}{2}$ state of Figure 8,

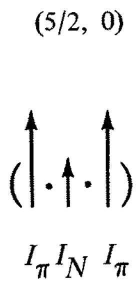

Figure 8–Interaction diagram M_8.

$$M_8 \approx 1565 \text{ MeV} \tag{6.1}$$

compares with the $N_{\frac{5}{2}}(1565)$ state, while Figure 9 gives

$$M_{9A,B} \approx [1690, 1915] \text{ MeV} \tag{6.2}$$

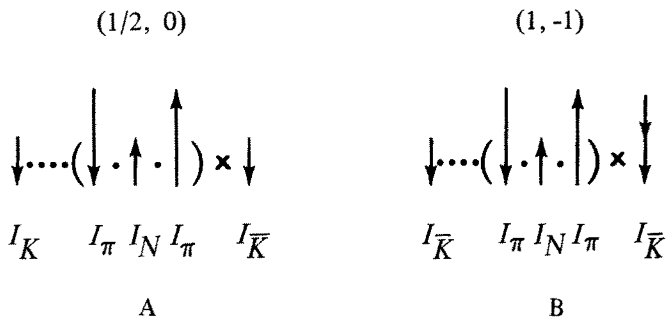

Figure 9–Interaction diagrams $M_{9A,B}$.

which compare well with the observed $N_{\frac{1}{2}}(1688)$ and $Y_1(1915)$ states. In association with the $Y_0(1405)$ state, Figure 10 gives

$$M^*_{10}(\bar{K}) \approx 1510 \text{ MeV} \tag{6.3}$$

to be compared with the observed $Y_0(1520)$ state.

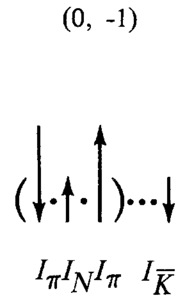

Figure 10–Interaction diagram M^*_{10}.

Baryon states with n_η-eta mesons (having zero isotopic spin) have a simple mass level formula,

$$M_B(\eta) \approx M_B + n_\eta(M_\eta + e_\eta) \tag{6.4}$$

For instance, the one-eta meson baryon state diagrams of Figure 11, where Θ represents the η meson, yield

$$M_{11A,B,C} \approx [1515, 1690, 1770] \text{ MeV} \tag{6.5}$$

to be compared with the $N_{\frac{1}{2}}(1515)$, $Y_0(1680)$ and $Y_1(1750)$ states, respectively.

It was noted above that the K-$\bar{K}$ meson pair in the M_{4A-4} and M_5 states might convert to a ρ meson, transforming the states, respectively, to the M_{12A} and M_{12B} states. The corresponding mass levels are

$$M_B(\rho) \approx M_N + F_{N\pi} + G_{N\pi\rho}$$

where

$$G_{N\pi\rho} \approx M_\rho + e_\rho + 2\,\vec{I}_{N\pi} \cdot \vec{I}_\rho f_1 \tag{6.6}$$

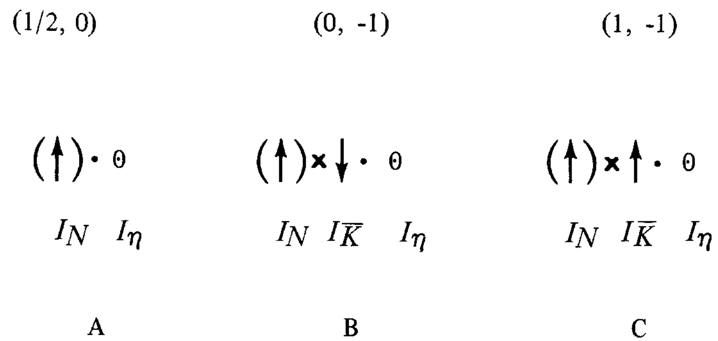

Figure 11–Interaction diagrams $M_{11A,B,C}$.

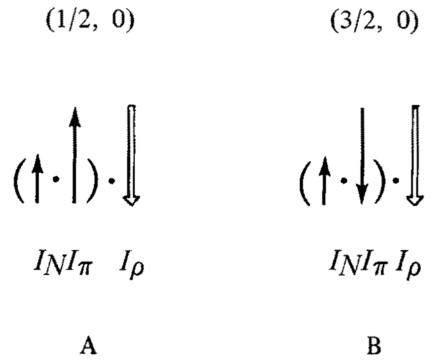

Figure 12—Interaction diagrams $M_{12A,B}$.

yielding,

$$M_{12A,B} \approx [1855, 1640] \text{ MeV} \tag{6.7}$$

They are indeed in good agreement with the observed $N_{\frac{1}{2}}(1860)$ and $N_{\frac{3}{2}}(1640)$ states.

7. $L(J)^P$ *of the Baryon States*

The following notations are now introduced:

$$\vec{L}_\pi = \Sigma(\vec{l}_\pi)_{n_\pi}, \qquad \vec{L}_{K,\bar{K}} = \Sigma(\vec{l}_{K,\bar{K}})_{n_{K,\bar{K}}}$$

$$\vec{J}_1 = \vec{J}_N + \vec{L}_\pi + \vec{L}_{\bar{K}}, \qquad \vec{J} = \vec{J}_1 + \vec{L}_K, \qquad \text{and} \qquad \vec{L} = \vec{L}_\pi + \vec{L}_{\bar{K}} + \vec{L}_K \tag{7.1}$$

where l_π and $l_{K,\bar{K}}$ are pion and $(K,\bar{K})$ meson orbital angular momenta, while J_N = nucleon core spin and p = baryon state parity.

The couplings of angular momenta presumably affect the baryon mass level no more strongly than those other terms that are neglected in this paper. General observation reveals that the following rules are operative:

(a) The L and $(J)^P$ of a baryon state are not affected by an involvement of quasidynamically interacting $\bar{K}$ mesons.
(b) A π-N interacts in a P-wave, yielding $(J)^P = (\frac{3}{2})^+$ and $(\frac{1}{2})^+$, respectively, for the cases of $I = \frac{3}{2}$ and $\frac{1}{2}$.
(c) In the $S \neq 0$ baryon state, $L \approx L_K$ general.

A possible physical implication of rule (a) for the quasidynamical interaction that induces a parity change and exhibits generally consistent results was previously discussed by Suh (1970). The $(I,J)^P = (\frac{1}{2},\frac{1}{2})^+$ π–N interaction of rule (b) is plausible, because the assignment not only gives generally consistent results but is also symmetric to the $(\frac{3}{2},\frac{3}{2})^+$ state counterpart.

As the mass levels of the baryon states increase, L generally increases. Exceptions arise, as discussed below, for those baryon states whose internal structure either transfigures or relates to the $\Delta l_K = -$ (b) rule. An interesting consequence of rule (c), then, is that the higher L-waves observed in the higher mass levels of the $S \neq 0$ baryon states are mainly due to the increasingly larger l_K.

It is observed for the ($S = 1, S_B = 0$) baryon states that "when the $\vec{I}_K$ vector that is parallel either to the $\vec{I}_N$ in the zero-pion baryon states or to the $\vec{I}_{N\pi}$ in the one-pion baryon states when $I_{N\pi} = \frac{3}{2}$ (or $I_{N\pi\bar{K}} = 1$, when $I_{N\pi} = \frac{1}{2}$), is flipped, the state is affected by $\Delta l_K = -2$ from the original state," thus producing the low angular momentum states in a comparatively high-energy domain. Because this $\vec{I}_K$ vector-flip reduces the energy of the states, it is not surprising that the process induces a reduction in the l_K quantum number. The avoidance of $\Delta l_K = -$ (a) rule here seems to indicate a strong inclination to preserve parity between the transition.

This $\Delta l_K = -$ (b) rule may not prevail, as observed, for the $S = 1$ hyperon states, because there are now at least two $\bar{K}$ mesons interacting in the states, and thus the lowering of l_K would greatly increase the probability of annihilating the K meson with one of the $\bar{K}$ mesons. This is another trait that vindicates the premises upon which the theory was developed.

One of those affected by the $\Delta l_K = -$ (b) rule is the M_{4A-5} state [the $\vec{I}_K$ vector flipped from the $F(\frac{7}{2})^+N_{\frac{3}{2}}(1920)/M_{4A-3}$], yielding the $P(\frac{1}{2})^+N_{\frac{3}{2}}(1910)$ state. Also affected are the M^*_{7B} and M_{2A-1} states [the $\vec{I}_K$ vector-flipped from the $D(\frac{3}{2})^-N_{\frac{3}{2}}(1690)/M^*_{7A}$ and $D(\frac{5}{2})^-N_{\frac{1}{2}}(1550)/M_{2A}$, respectively], yielding, respectively, $S(\frac{1}{2})^-N_{\frac{1}{2}}(1690)$ and $S(\frac{1}{2})^-N_{\frac{1}{2}}(1550)$ states at a comparatively high-energy domain for the S-wave interaction to arise.

The general validity of the mass level formula (2.7), as observed throughout this paper (or, specifically, the small mass level difference, for instance, between the $N_{\frac{3}{2}}(1920) \mid l_K = 3$ and $N_{\frac{3}{2}}(1910) \mid l_K = 1$ states), is sufficient evidence to generalize and uphold the relevance of the energy relation, i.e., equation (2.3).

It has also been noted throughout this paper that participation of a K meson in the formation of baryon states requires elusion of its outright annihilation with a $\bar{K}$ meson, by interacting in a higher orbital than those of the $\bar{K}$ mesons. All of the $S \neq 0$ baryon states considered in this paper, where necessarily $l_{\bar{K}} = 0$ for the quasidynamically interacting $\bar{K}$ mesons, the l_K is nonzero except for the states $S(\frac{1}{2})^-N_{\frac{1}{2}}(1550)/M_{2A-1}$ and $S(\frac{1}{2})^-N_{\frac{1}{2}}(1690)/M^*_{7B}$ that are affected by the $\Delta l_K = -$ (b) rule.

The possible K-$\bar{K}$ meson annihilation in the M_{2A-1} and M^*_{7B} states leaves a physical nucleon, while the omission of the K meson reduces the states, respectively, to the Λ and Σ states. Therefore, even if the K meson risks itself for possible annihilation with the quasidynamically interacting $\bar{K}$ meson by interacting in an S-wave, a certain degree of stability may remain intact for them to be observable as the $S(\frac{1}{2})^-N_{\frac{1}{2}}(1550)$ and $S(\frac{1}{2})^-N_{\frac{1}{2}}(1690)$ states.

The situation in the M_{4A} state is different. The $\vec{I}_K$ vector-flip in the $D(\frac{3}{2})^-N_{\frac{1}{2}}(1520)/M_{4A-2} \mid l_K = 2$ state, gives $l_K = 0$ for the M_{4A} state. How-

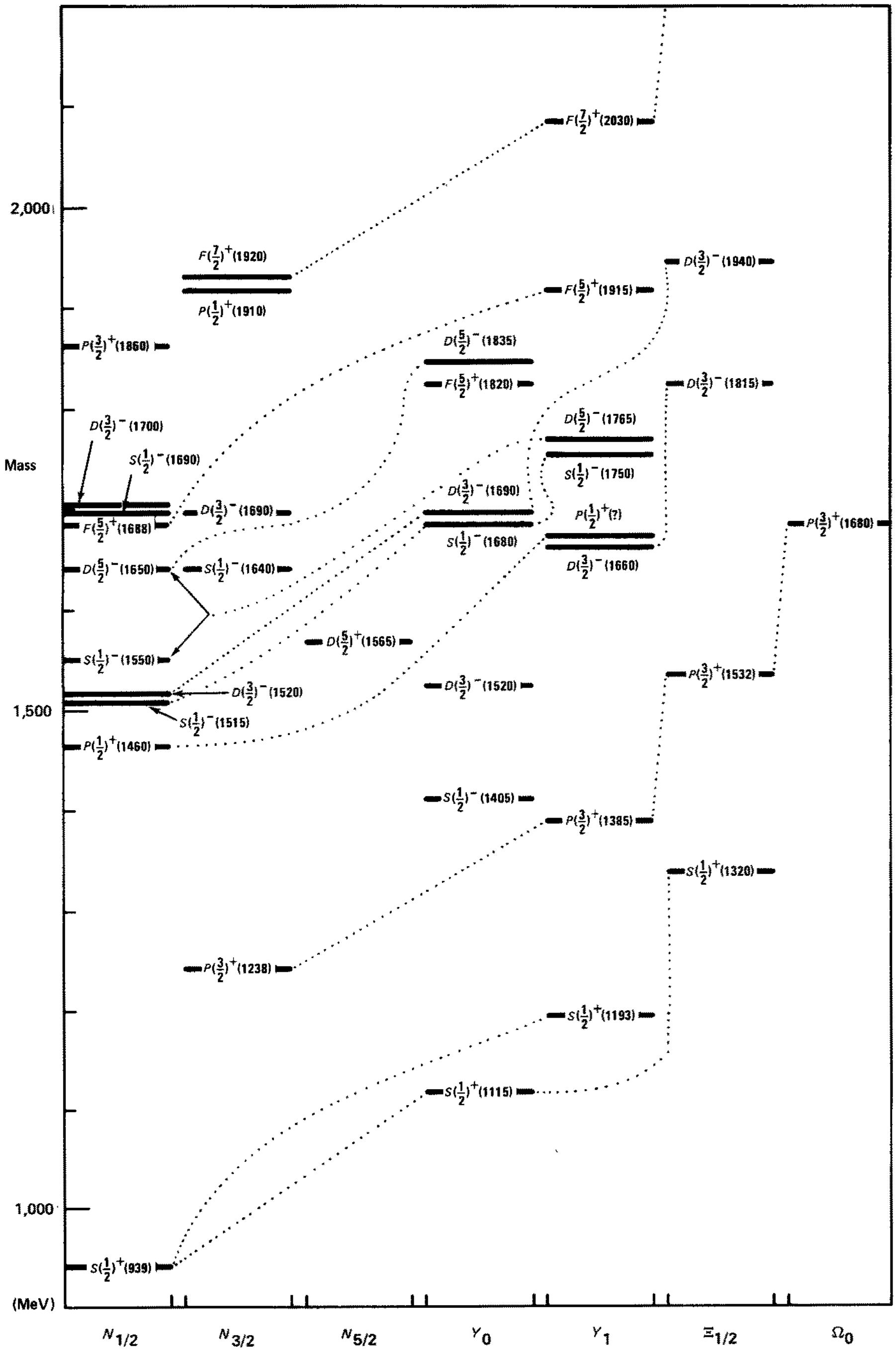

Figure 13—Baryon mass spectra with their probable $L(J)^P$.

ever, both the possible K-$\bar{K}$ meson annihilation and omission of the K meson in the M_{4A} state reduce it to nonexistent states, thus disfavoring, as noted in the Table, the production of the resulting $P(\frac{1}{2})^- N_{\frac{1}{2}}(1520)/M_{4A}$ state.

The excited $\bar{K}$ meson in the $M^*_{4A-2}(\bar{K})$ state interacts in an $l_{\bar{K}} = 1$ wave, thus driving l_K to 2 to produce a $D(\frac{3}{2})^-$ state. The prospect of outright K-$\bar{K}$ meson annihilation disfavors the M_{2A-2} state, to be produced instead in the $M^*_{7B}(K)$ state.

The mass levels of the baryon states with their $L(J)^P$ assignments as discussed in this paper are shown in Figure 13. Particle states of various multiplets are linked by dotted lines. Note here that although the M_{2A-1} state is, due to the $\Delta l_K = -$ (b) rule, produced as the $S(\frac{1}{2})^- N_{\frac{1}{2}}(1550)$, it actually belongs to the $[L = 2, p = (-)]$ multiplicity as for the M_{2B-1} state that, by overlapping with the M_{4B} state, produces the strong resonance $D(\frac{5}{2})^- Y_1(1765)$. The solid arrows pointing to the $N_{\frac{1}{2}}(1650)$ and $N_{\frac{1}{2}}(1550)$ states thus indicate that the latter actually belongs to the $D(\frac{5}{2})^-$ multiplet. It is now clear that the following rule is in effect: "the baryon states in a multiplet possess the same $L(J)^P$ configuration."

The theory then predicts, for instance, two closely overlapping Y_1 states from the two multiplets, one in $D(\frac{3}{2})^-$ and the other in $P(\frac{1}{2})^+$, around the 1660 MeV mass level. The existence of a $D(\frac{3}{2})^- Y_1(1660)$ state has been definitely confirmed experimentally, while observational evidences are emerging for a possible overlapping of a $P(\frac{1}{2})$ Y_1 state there.

The nature of dynamically interacting $\bar{K}$ and K mesons should be comparable in the determination of $L(J)^P$. Thus, the excited $\bar{K}$ meson in the $Y_0(1520)$ state interacts in an $l_{\bar{K}} = 2$ wave, while it interacts in the $l_{\bar{K}} = 0$ wave in the $Y_0(1405)$ state. It was stated above that the $l_{\bar{K}} = 1$ interaction is in effect in the $M^*_{4A-2}(\bar{K})$ state.

The η-meson baryon states $M_{11A,B,C}$ are observed in the S-states[8], i.e., $S(\frac{1}{2})^- N_{\frac{1}{2}}(1515)$, $S(\frac{1}{2})^- Y_0(1680)$, and $S(\frac{1}{2})^- Y_1(1750)$. The K-K meson pair in the M_{4A-4} and M_5 states (affected by the Anomaly I and IA) appear to be transformed to a ρ-meson, consequently, even though the resultant $M_{12A,B}$ states, i.e., $N_{\frac{1}{2}}(1860)$ and $N_{\frac{3}{2}}(1640)$, lie in the high energy domain, they are produced in the low P and S states, respectively. Analysis of the $N_{\frac{3}{2}}(1640)$ state indicates that the nature of angular momentum of the ρ-meson is orbital, in line with the conjecture that the ρ-meson is a K-$\bar{K}$ resonance state.

[8] There are two adjacent $S(\frac{1}{2})^- N_{\frac{1}{2}}$ states, i.e., $N_{\frac{1}{2}}(1550)/M_{2A-1}$ and $N_{\frac{1}{2}}(1515)/M_{11A}$, whose average mass level is 1535 MeV. Therefore, the single state $S(\frac{1}{2})^- N_{\frac{1}{2}}(1535)$ often empirically quoted (see, for instance, 1974 issue of "Review of Particle Properties", *Physics Letters*, **50B**, 1), despite ambiguities, probably represents the overall effect of these two adjacent $S(\frac{1}{2})^- N_{\frac{1}{2}}$ states. The $N_{\frac{1}{2}}(1550)/M_{2A-1}$ state (produced without involving η-meson interaction) would scarcely decay into the $N\eta$ mode. The observed 55% of the $N\eta$ decay mode of the empirically quoted $N_{\frac{1}{2}}(1535)$ state then means that the $N_{\frac{1}{2}}(1515)/M_{11A}$ state (produced solely by the N-η resonance) almost exclusively decays into the $N\eta$ mode itself. The strong N-η interaction thus resonates elastically in an S-wave.

8. *Conclusion*

The logic that helped develop the theory introduces a unique structure for the strongly interacting particle states. Increasingly coherent interaction mechanisms emerge to form the particle states, and rules that are unmistakably anticipatory evolve to govern them. The evidence that support the importance of evading the outright K-$\bar{K}$ pair annihilation in the formation of baryon states, for instance, is compelling and multitudinous.

Since the theory was proposed in 1959 when only a few particle states were known, its outlook has steadily improved and the striking one-to-one agreement achieved between the prediction and observation is impressive. Moreover, it is possible to look into the workings of the truly wondrous particle structures.

Briefly note here that the meson resonances satisfy the mass relation,

$$M_m \approx [M_B + (M_{K,\bar{K}})] - M_b \tag{8.1}$$

in which the $(M_{K,\bar{K}})$ term is optional and M_b represents the mass of any one of the $N_{\frac{1}{2}}(939)$, $N_{\frac{3}{2}}(1238)$, $\Lambda_0(1115)$ and $\Xi_{\frac{1}{2}}(1320)$ state. The M_B represents the mass of any one of the well established baryon states or those that are disfavored to form because of the condition (2.8) and/or Anomalies I and IA. Examination of other related properties should yield further insights into the strongly interacting particle physics.

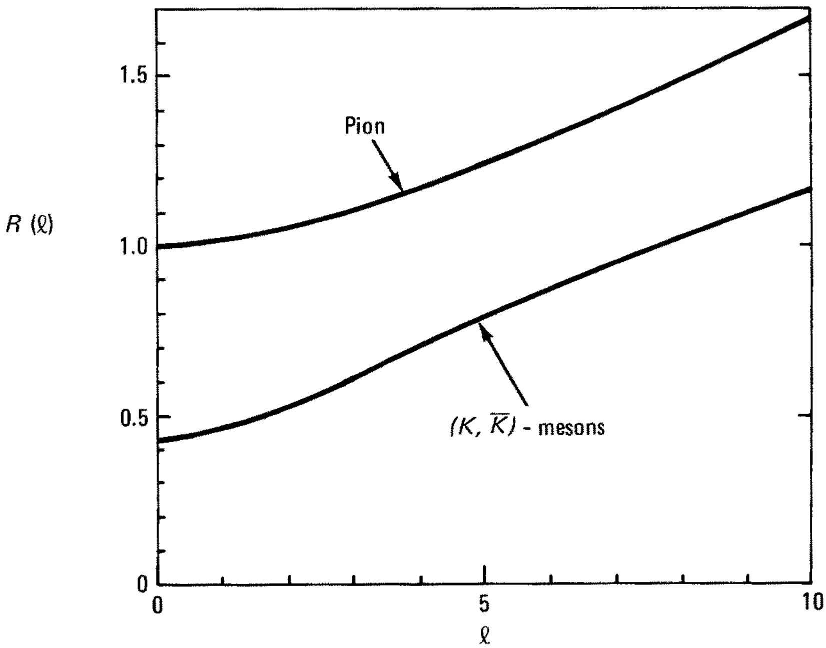

Figure 14—Expansible interaction radii $R(l)$ for the pion and $(K, \bar{K})$ mesons, in units of the pion Compton wavelength.

The exploratory work presented in this paper is subject to modification as more understanding of the phenomena develops. Nevertheless, this author is convinced that light is being seen at the end of the long arduous search to uncover the truly wondrous workings of strongly interacting particle physics.

Appendix

Expansible Potential. An intrinsic uncertainty created by the virtual emission field between the nucleon core and a meson interacting in high partial waves may become significant to partially modulate, for instance, the role of the centrifugal force. The potential around the effective interaction radius and outward may then be approximated in a modified Yukawa potential form, while it should be repulsive toward the core. Pending a more rigorous development, however, the following simple potential energy that may generalize the energy relation (2.3) will be conjectured:

$$V_l(r) \approx -g^2 e^{-\mu(l)r}/r \tag{A.1}$$

where $\mu(l = 0) = \mu$. Then, from the Schrödinger equation

$$\left[-\frac{d^2}{dr^2} + \frac{(l+\frac{1}{2})^2 - \frac{1}{4}}{r^2} + 2\mu\{V_l(r) - E\}\right] U = 0 \tag{A.2}$$

$$\frac{d(l+\frac{1}{2})^2}{2\mu} \int \frac{U^2}{r^2}\, dr \approx dE \int U^2\, dr - \int d\mu(l)\, \mathcal{T}\, |rV_l(r)|\, U^2\, dr \tag{A.3}$$

where $\mathcal{T} = 1 + (2/\mu(l)r)$ when incorporated with $\mu g \approx \mu_K g_K \approx \mu_\pi g_\pi$ (Suh, 1970).

For $d\mu(l) \approx 0$, as adapted for the Regge families of particle states,

$$\Delta E \approx \Delta(l+\tfrac{1}{2})^2 \frac{\int (U^2/r^2)\, dr}{2\mu \int U^2\, dr} \tag{A.4}$$

giving, without ever being borne out experimentally, large mass level shifts among various l-states. For $dE \approx 0$, as observed throughout this paper to generalize and uphold the energy relation (2.3),

$$d(l+\tfrac{1}{2})^2 \approx -2\mu \frac{\int d\mu(l)\, \mathcal{T}\, |rV_l(r)|\, U^2\, dr}{\int (U^2/r^2)\, dr} \tag{A.5}$$

yielding the expansible effective interaction radius of Figure 14, i.e.,

$$R(l) \approx 1/\mu(l) \approx \left[1 + (\mu/\mu_\pi)^2 \frac{l(l+1)}{30}\right]^{\frac{1}{3}} / \mu \tag{A.6}$$

References

Okubo, S. (1962). *Progress of Theoretical Physics*, **27**, 947.
Suh, K. S. (1960). *Bulletin American Phys. Soc.* **5**, 255.
Suh, K. S. (1968). *Inf. zur Kernforschung und Kerntechnik*, **11-8302**, 137.
Suh, K. S. (1970). *International Journal of Theoretical Physics*, **3**, 359.